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三维准地转运动的非线性稳定性研究的一个注记*

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关键词　三维准地转运动、非线性稳定性

文献[1]对于水平区域为 β-平面上有界区域的三维准地转运动，首次建立了只需确定两维 Laplace 算子最小特征值的非线性稳定性判据，它对应于 Arnold 第二定理[1,2]. 本文通过更加精细的分析、估计，给出了一个新的判据，它优于文献[1]中的结果.

在 β-平面近似下，三维准地转模式如下[3,4]:

$$\frac{\partial q}{\partial t} + J(\psi, q) = 0, \text{在 } \Omega \times (0, \infty), \tag{1}$$

$$\frac{\partial b_i}{\partial t} + J(\psi, b_i)|_{z=z_i} = 0, \text{在 } D \text{ 中}, \ i = 0, 1, \tag{2}$$

这里 ψ, q 分别表示流函数与位势涡度.

$$q = \nabla^2\psi + \frac{f_0^2}{\rho_0}\frac{\partial}{\partial z}\left(\frac{\rho_0}{N^2}\frac{\partial\psi}{\partial z}\right) + f, \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2},$$

$$J(\psi, q) = \frac{\partial\psi}{\partial x}\frac{\partial q}{\partial y} - \frac{\partial\psi}{\partial y}\frac{\partial q}{\partial x}, \ b_1 = \frac{\partial\psi}{\partial z}\Big|_{z=z_1}, \ b_0 = \left(\frac{\partial\psi}{\partial z} + \frac{N^2}{f_0}\eta\right)\Big|_{z=z_0},$$

$\rho_0(z)$, $N(z)$ 分别是密度与 Brunt-Väisälä 频率；$f = f_0 + \beta y$ 是柯氏参数；f_0, β 为常数；$\eta(x, y)$ 是地形函数；D 为 β-平面上有界多连通(或单连通)域，边界 $\partial D = \bigcup_{j=0}^{J} \partial D_j$ 充分光滑，∂D_0 是外边界. $\Omega = D \times (z_0, z_1)$，$0 \leqslant z_0 < z_1 < \infty$. 此外，侧边界条件为[3,4]

$$\frac{\partial\psi}{\partial\tau}\Big|_{\partial D_j} = 0, \frac{\partial}{\partial t}\int_{\partial D_j} \nabla\psi \cdot \boldsymbol{n} ds = 0, \ j = 0, \cdots, J, \tag{3}$$

这里 $\frac{\partial}{\partial\tau}$ 为 ∂D 的切向求导算子，$\boldsymbol{n}$ 为 ∂D 的单位外法向量.

考察(1)—(3)式的定常态 $(\bar\psi(x,y,z), \bar q(x,y,z))$，它满足[1]

$$\bar\psi(x,y,z) = P(\bar q(x,y,z), z), \text{在 } \Omega \text{ 中}, \tag{4}$$

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$$\bar{\psi}(x,y,z_i)=Q_i(\bar{b}_i),\ \text{在}D\text{中},\ i=0,1, \tag{5}$$

这里 $\bar{b}_i$ 由 b_i 定义中置 ψ 为 $\bar{\psi}$ 而得到. $P(\xi,z)$, $Q_i(\xi)$ 是任何三个一阶连续可导函数, 满足[1]

$$\begin{aligned}
&0<c_1\equiv\min_{\Omega}(-\nabla\bar{\psi}/\nabla\bar{q})\leqslant-\partial P/\partial\xi\leqslant\max_{\Omega}(-\nabla\bar{\psi}/\nabla\bar{q})\equiv c_2<\infty,\\
&0<c_{11}\equiv\min_{D}(\nabla\bar{\psi}|_{z=z_1}/\nabla\bar{b}_1)\leqslant dQ_1/d\xi\leqslant\max_{D}(\nabla\bar{\psi}|_{z=z_1}/\nabla\bar{b}_1)\equiv c_{21}<\infty,\\
&0<c_{10}\equiv\min_{D}(-\nabla\bar{\psi}|_{z=z_0}/\nabla\bar{b}_0)\leqslant-dQ_0/d\xi\leqslant\max_{D}(-\nabla\bar{\psi}|_{z=z_0}/\nabla\bar{b}_0)\equiv c_{20}<\infty,
\end{aligned} \tag{6}$$

这里 $\nabla\bar{\psi}/\nabla\bar{q}\equiv\dfrac{\partial\bar{\psi}}{\partial x}\Big/\dfrac{\partial\bar{q}}{\partial x}\equiv\dfrac{\partial\bar{\psi}}{\partial y}\Big/\dfrac{\partial\bar{q}}{\partial y}$,余类推.

为简单计,本文仅考虑初值扰动,但结果适用于初值与参数的扰动[1]. 记

$$F(\xi,z)=\int^{\xi}P(\tau,z)d\tau,\ G_i(\xi)=\int^{\xi}Q_i(\tau)d\tau, \tag{7}$$

$$H_3(t)=\frac{1}{2}\int_{\Omega}\rho_0\left(|\nabla(\psi-\bar{\psi})|^2+\frac{f_0^2}{N^2}\left|\frac{\partial}{\partial z}(\psi-\bar{\psi})\right|^2\right)dxdydz,$$

$$\begin{aligned}
H_4(t)=&\int_{\Omega}\rho_0[F(q,z)-F(\bar{q},z)-P(\bar{q})(q-\bar{q})]dxdydz\\
&+\sum_{i=0}^{1}(-1)^i\int_D\rho_0(z_i)\frac{f_0^2}{N^2(z_i)}[G_i(b_i)-G_i(\bar{b}_i)-Q_i(\bar{b}_i)(\bar{b}_i-b_i)]dxdy.
\end{aligned}$$

由文献[1]易得

$$\frac{d}{dt}(H_3(t)+H_4(t))\equiv0,t\geqslant0. \tag{8}$$

由(6),(8)式及 Taylor 公式,得

$$\begin{aligned}
&\frac{c_1}{2}\int_{\Omega}\rho_0|q-\bar{q}|^2dxdydz+\sum_{i=0}^{1}\frac{c_{1i}}{2}\int_D f_0^2\frac{\rho_0(z_i)}{N^2(z_i)}(b_i-\bar{b}_i)^2dxdy\\
&\leqslant H_3(t)-H_3(0)-H_4(0).
\end{aligned} \tag{9}$$

为了估计 $H_3(t)$, 记 $\mathrm{mes}(D)=\int_D dxdy$. $\varphi(z,t)\equiv\int_D(\psi_0-\psi)dxdy/\mathrm{mes}(D)$,这里 $\psi_0=\psi(x,y,z,0)$. 易见 $\tilde{\psi}\equiv\psi+\varphi$ 满足

$$\frac{d}{dt}\int_D\tilde{\psi}dxdy\equiv0,t\geqslant0,z\in[z_0,z_1]. \tag{10}$$

再由式(1)—(3)即得

$$\frac{d}{dt}\int_D\frac{\partial}{\partial z}\left(\frac{\rho_0}{N^2}\frac{\partial\psi}{\partial z}\right)dxdy\equiv0,t\geqslant0,z\in[z_0,z_1]. \tag{11}$$

$$\frac{d}{dt}\int_D\frac{\partial\psi}{\partial z}\Big|_{z=z_i}dxdy\equiv0,t\geqslant0,i=0,1. \tag{12}$$

$$\frac{\partial}{\partial z}\left(\frac{\rho_0}{N^2}\frac{\partial\varphi}{\partial z}\right)\equiv0,\frac{\partial\varphi}{\partial z}\Big|_{z=z_i}\equiv0,i=0,1. \tag{13}$$

$$\tilde{q}\equiv\nabla^2\tilde{\psi}+\frac{f_0^2}{\rho_0}\frac{\partial}{\partial z}\left(\frac{\rho_0}{N^2}\frac{\partial\tilde{\psi}}{\partial z}\right)+f=q. \tag{14}$$

$$b_1 = \frac{\partial\tilde{\psi}}{\partial z}\bigg|_{z=z_1},\ b_0 = \left(\frac{\partial\tilde{\psi}}{\partial z} + \frac{N^2}{f_0}\eta\right)\bigg|_{z=z_0}. \tag{15}$$

$$\tilde{q} - \bar{q} = \nabla^2(\tilde{\psi} - \bar{\psi}) + \frac{f_0^2}{\rho_0}\frac{\partial}{\partial z}\left(\frac{\rho_0}{N^2}\frac{\partial}{\partial z}(\tilde{\psi} - \bar{\psi})\right).$$

将上式乘以 $(\tilde{\psi} - \bar{\psi},)$分部积分,利用式(3),得

$$\int_\Omega \rho_0|\nabla(\tilde{\psi} - \bar{\psi})|^2 dxdydz + \int_\Omega f_0^2\frac{\rho_0}{N^2}\left|\frac{\partial}{\partial z}(\tilde{\psi} - \bar{\psi})\right|^2 dxdydz$$

$$-\int_D f_0^2\left[\frac{\rho_0}{N^2}(\tilde{\psi} - \bar{\psi})\frac{\partial}{\partial z}(\tilde{\psi} - \bar{\psi})\right]\bigg|_{z=z_0}^{z=z_1}$$

$$-\sum_{j=0}^{J}\int_{z_0}^{z_1}\rho_0(\tilde{\psi} - \bar{\psi})\bigg|_{\partial D_j}dz\int_{\partial D_j}\nabla(\psi_0 - \bar{\psi})\cdot\boldsymbol{n}ds$$

$$= -\int_\Omega \rho_0(\tilde{q} - \bar{q})(\tilde{\psi} - \bar{\psi})dxdydz. \tag{16}$$

反复分部积分并利用(13)式可得

$$\int_\Omega f_0^2\frac{\rho_0}{N^2}\left|\frac{\partial}{\partial z}(\tilde{\psi} - \bar{\psi})\right|^2 dxdydz = \int_\Omega f_0^2\frac{\rho_0}{N^2}\left|\frac{\partial}{\partial z}(\psi - \bar{\psi})\right|^2 dxdydz.$$

综合上式与(16)式得

$$H_3(t) \leqslant \frac{\varepsilon}{4}\int_\Omega \rho_0|q - \bar{q}|^2 dxdydz + \frac{1}{4\varepsilon}\int_\Omega \rho_0|\tilde{\psi} - \bar{\psi}|^2 dxdydz$$

$$+\sum_{i=0}^{1}\frac{\varepsilon_i}{4}\int_D f_0^2\frac{\rho_0(z_i)}{N^2(z_i)}(b_i - \bar{b}_i)^2 dxdy + \sum_{i=0}^{1}\frac{1}{4\varepsilon_i}\int_D f_0^2\frac{\rho_0(z_i)}{N^2(z_i)}|(\tilde{\psi} - \bar{\psi})(z_i)|^2 dxdy$$

$$+ |H_*(t)|,$$

这里 $\varepsilon, \varepsilon_0, \varepsilon_1$ 为待定正常数, $H_*(t)$ 是式(16)左端的最后一项.

记 λ_2 是椭圆边值问题.

$\nabla^2 u + \lambda u = 0$, 在$D$ 中, $\dfrac{\partial u}{\partial \tau}\bigg|_{\partial D_j} = 0, \int_{\partial D_j}\nabla u\cdot\boldsymbol{n}ds = 0$, $j = 0,\cdots,J$ 的最小正特征值. 因为 $\int_D(\tilde{\psi} - \psi_0)dxdy = 0, \int_{\partial D_j}\nabla(\psi_0 - \tilde{\psi})\cdot\boldsymbol{n}ds = 0, j = 0,\cdots J$; 由偏微分方程理论,可得

$$\int_D|\tilde{\psi} - \psi_0|^2 dxdy \leqslant \frac{1}{\lambda_2}\int_D|\nabla(\psi - \psi_0)|^2 dxdy. \tag{17}$$

利用上式,再注意到能量守恒

$$\frac{\partial}{\partial t}\int_\Omega \rho_0\left(|\nabla\psi|^2 + \frac{f_0^2}{N^2}\left|\frac{\partial\psi}{\partial z}\right|^2\right)dxdydz = 0,$$

并利用 Hölder 不等式 $\int_\Omega|ab|dxdydz \leqslant \left(\int_\Omega a^2 dxdydz\right)^{\frac{1}{2}}\cdot\left(\int_\Omega b^2 dxdydz\right)^{\frac{1}{2}}$,可得

$$\int_\Omega \rho_0|\tilde{\psi} - \bar{\psi}|^2 dxdydz \leqslant \frac{1}{\lambda_2}\int_\Omega \rho_0|\nabla(\psi - \bar{\psi})|^2 dxdydz + H_{**}, \tag{18}$$

这里

$$H_{**}=\frac{1}{\lambda_2}\int_\Omega \rho_0|\nabla(\phi-\phi_0)|^2\mathrm{d}x\mathrm{d}y\mathrm{d}z+\frac{2\sqrt{2}}{\lambda_2}\left(\int_\Omega \rho_0|\nabla(\phi_0-\bar{\phi})|^2\mathrm{d}x\mathrm{d}y\mathrm{d}z\right)^{\frac{1}{2}}$$
$$\cdot\left[\int_\Omega \rho_0\left(|\nabla\phi_0|^2+\frac{f_0^2}{N^2}\left|\frac{\partial\phi_0}{\partial z}\right|^2+|\nabla\bar{\phi}|^2\right)\mathrm{d}x\mathrm{d}y\mathrm{d}z\right]^{\frac{1}{2}}+\int_\Omega \rho_0|\phi_0-\bar{\phi}|^2\mathrm{d}x\mathrm{d}y\mathrm{d}z$$
$$+\frac{2\sqrt{2}}{\lambda_2^{\frac{1}{2}}}\left(\int_\Omega \rho_0|\phi_0-\bar{\phi}|^2\mathrm{d}x\mathrm{d}y\mathrm{d}z\right)^{\frac{1}{2}}$$
$$\cdot\left[\int_\Omega \rho_0\left(2|\nabla\phi_0|^2+\frac{f_0^2}{N^2}\left|\frac{\partial\phi_0}{\partial z}\right|^2\right)\mathrm{d}x\mathrm{d}y\mathrm{d}z\right]^{\frac{1}{2}}.$$

利用文献[1]的(2.19)式,我们有

$$\int_D f_0^2\frac{\rho_0(z_i)}{N^2(z_i)}|(\tilde{\phi}-\bar{\phi})(z_i)|^2\mathrm{d}x\mathrm{d}y\leqslant F(\chi)\int_\Omega \rho_0|\tilde{\phi}-\bar{\phi}|^2\mathrm{d}x\mathrm{d}y\mathrm{d}z$$
$$+\chi\int_\Omega \frac{f_0^2\rho_0}{N^2}\left|\frac{\partial}{\partial z}(\tilde{\phi}-\bar{\phi})\right|^2\mathrm{d}x\mathrm{d}y\mathrm{d}z, \tag{19}$$

这里$\chi>0$是待定常数,而

$$F(\chi)=F_1+F_2/\chi,\ F_1=\max_{[z_0,z_1]}f_0^2\left[\frac{\rho_0}{|z_1-z_0|N^2}+\left|\frac{\partial}{\partial z}\left(\frac{\rho_0}{N^2}\right)\right|\right]\Big/\min_{[z_0,z_1]}\rho_0,$$
$$F_2=f_0^2\cdot\max_{[z_0,z_1]}\frac{1}{N^2}.$$

综合式(17)—(19)得

$$H_3(t)\leqslant\frac{\varepsilon}{4}\int_\Omega \rho_0|q-\bar{q}|^2\mathrm{d}x\mathrm{d}y\mathrm{d}z+\frac{1}{4\lambda_2}\left(\frac{1}{\varepsilon}+\frac{F(\chi)}{\varepsilon_0}+\frac{F(\chi)}{\varepsilon_1}\right)\int_\Omega \rho_0|\nabla(\phi$$
$$-\bar{\phi})|^2\mathrm{d}x\mathrm{d}y\mathrm{d}z+\frac{\chi}{4}\left(\frac{1}{\varepsilon_0}+\frac{1}{\varepsilon_1}\right)\int_\Omega \frac{f_0^2\rho_0}{N^2}\left|\frac{\partial}{\partial z}(\phi-\bar{\phi})\right|^2\mathrm{d}x\mathrm{d}y\mathrm{d}z$$
$$+\sum_{i=0}^{1}\frac{\varepsilon_i}{4}\int_D f_0^2\frac{\rho_0(z_i)}{N^2(z_i)}(b_i-\bar{b}_i)^2\mathrm{d}x\mathrm{d}y+|H_*(t)|$$
$$+\frac{1}{4}\left(\frac{1}{\varepsilon}+\frac{F(\chi)}{\varepsilon_0}+\frac{F(\chi)}{\varepsilon_1}\right)H_{**}.$$

若存在正数 $\varepsilon,\varepsilon_0,\varepsilon_1$ 与 χ,使

$$M=\min\left\{1-\frac{1}{2\varepsilon\lambda_2}-\frac{F(\chi)}{2\varepsilon_0\lambda_2}-\frac{F(\chi)}{2\varepsilon_1\lambda_2},1-\frac{\chi}{2\varepsilon_0}-\frac{\chi}{2\varepsilon_1}\right\}>0, \tag{20}$$

则可得

$$H_3(t)\leqslant\frac{\varepsilon}{4M}\int_\Omega \rho_0|q-\bar{q}|^2\mathrm{d}x\mathrm{d}y\mathrm{d}z+\sum_{i=0}^{1}\frac{\varepsilon_i}{4M}\int_D f_0^2\frac{\rho_0(z_i)}{N^2(z_i)}(b_i-\bar{b}_i)^2\mathrm{d}x\mathrm{d}y$$
$$+\frac{1}{M}|H_*(t)|+\frac{1}{4M}\left(\frac{1}{\varepsilon}+\frac{F(\chi)}{\varepsilon_0}+\frac{F(\chi)}{\varepsilon_1}\right)H_{**}. \tag{21}$$

记 $q_0=q(x,y,z,0),b_{i0}=b_i(x,y,z_i,0,),\bar{b}_{i0}=\bar{b}_i(x,y,z_i,0)$, 利用式(6),(9)及上式,得

$$\left(\frac{c_1}{2}-\frac{\varepsilon}{4M}\right)\int_\Omega \rho_0|q-\bar{q}|^2\mathrm{d}x\mathrm{d}y\mathrm{d}z+\sum_{i=0}^{1}\left(\frac{c_{1i}}{2}-\frac{\varepsilon_i}{4M}\right)\int_D f_0^2\frac{\rho_0(z_i)}{N^2(z_i)}(b_i-\bar{b}_i)\mathrm{d}x\mathrm{d}y$$

$$\leqslant \frac{c_2}{2}\int_\Omega \rho_0|q_0-\bar{q}|^2 dxdydz+\sum_{i=0}^{1}\frac{c_{2i}}{2}\int_D \rho_0(z_i)\frac{f_0^2}{N^2(z_i)}|b_{i0}-\bar{b}_{i0}|^2 dxdy$$

$$+\frac{1}{M}|H_*(t)|+\frac{1}{4M}\left(\frac{1}{\varepsilon}+\frac{F(\chi)}{\varepsilon_0}+\frac{F(\chi)}{\varepsilon_1}\right)H_{**}-H_3(0). \quad (22)$$

由文献[1]的附录中定理可知,存在正常数 $\varepsilon,\varepsilon_0,\varepsilon_1$ 与 χ 使得$M>0$ 且(22)式左端各项之系数为正的充分必要条件是

$$\left[\lambda_2-\left(\frac{1}{c_{10}}+\frac{1}{c_{11}}\right)F_1-\left(\frac{1}{c_{10}}+\frac{1}{c_{11}}\right)^2F_2\right]c_1>1. \quad (23)$$

若基本气流 $(\bar{\phi},\bar{q})$ 满足式(23),则可取定 $\varepsilon,\varepsilon_0,\varepsilon_1$ 与 χ,因而M之值亦给定,使$M>0$ 且式(22)左端各项系数为正,由式(21),(22)可知,欲证 $(\bar{\phi},\bar{q})$ 是非线性稳定的,只需说明,若初值 ϕ_0,q_0 满足

$$\int_\Omega \rho_0|\phi_0-\bar{\phi}|^2 dxdydz\to 0,\quad \int_\Omega \rho_0|q_0-\bar{q}|^2 dxdydz\to 0,$$

$$\int_\Omega \rho_0\left[|\nabla(\phi_0-\bar{\phi})|^2+\frac{f_0^2}{N^2}\left|\frac{\partial}{\partial z}(\phi_0-\bar{\phi})\right|^2\right]dxdydz\to 0, \quad (24)$$

$$\sum_{j=0}^{J}\int_{z_0}^{z_1}\left|\int_{\partial D}\nabla(\phi_0-\bar{\phi})\cdot \boldsymbol{n}ds\right|\to 0,\ \int_D |b_{i0}-\bar{b}_{i0}|^2 dxdy\to 0, i=0,1。$$

则 $H_{**}\to 0$ 且 $H_*(t)$ 关于 t 一致趋于零.

由 H_{**} 之表达式易见,当式(24)成立时, H_{**} 趋于零. 利用式(18),类似于文献[1]中式(2.27)的讨论,可以证明在条件式(24)下,$H_*(t)$ 关于 t 一致地趋于零.

综合上述讨论,我们得非线性稳定性判据. 若基本气流 $(\bar{\phi},\bar{q})$ 满足式(4)—(6)且式(23)成立,则它是非线性稳定的. 即,对任何 $\Delta_1>0,\Delta_2>0$, 存在正常数 $\delta_k(k=1,\cdots,6)$, 使得若(24)式各项分别小于 δ_k 时,恒有

$H_3(t)\leqslant\Delta_1, t\geqslant 0,$

$$\frac{c_1}{2}\int_\Omega \rho_0|q-\bar{q}|^2 dxdydz+\sum_{i=0}^{1}\frac{c_{1i}}{2}\int_D f_0^2\frac{\rho_0(z_i)}{N^2(z_i)}(b_i-\bar{b}_i)^2 dxdy\leqslant\Delta_2, t\geqslant 0.$$

现在我们来说明,上述判据优于文献[1]中的判据 2.2. 为此,只需证明 $\lambda_2\geqslant\lambda_1$ 恒成立,且存在区域 D,使 $\lambda_2>\lambda_1$ 成立(λ_1 之定义见下文).

事实上,若D是平行于 x 轴的周期带状区域,在 y 方向宽度为 Y_1,由文献[1]§3 的方法,可知 $\lambda_1=\frac{\pi^2}{4Y_1^2}<\frac{\pi^2}{Y_1^2}=\lambda_2$.

另一方面,因为 λ_1 是 $\left\{\nabla^2u+\lambda u=0,\ 在D中; u|_{\partial D_0}=0, \frac{\partial u}{\partial\tau}\Big|_{\partial D_j}=0, \int_{\partial D_j}\nabla u\cdot \boldsymbol{n}ds=0, j=1\cdots,J\right\}$ 的最小特征值,可知 $\lambda_1=\min\limits_{u\in X}\left(\int_D|\nabla u|^2dxdy\Big/\int_D u^2dxdy\right)$, 而 $\lambda_2=\min\limits_{u\in Y}\left(\int_D|\nabla u|^2dxdy\Big/\int_D u^2dxdy\right)$. 这里 $X=\{u|u$ 满足 λ_1 的定义中的边界条件, $u\neq 0\}$, $Y=\left\{u|u\right.$ 满足 λ_2 的定义中的边界条件, $\left.\int_D udxdy=0, u\neq 0\right\}$. 对任何 $u\in Y$, 若 $c_0=u|_{\partial D_0}=0$, 则

$u\in X$；若 $c_0\neq 0$，令 $v=u-c_0$. 易见 $v\neq 0$（否则与 $u\in Y$ 矛盾），又可验证 v 满足 $v|_{\partial\Gamma_0}=0$，故 $v\in X$. 因而 $\int_D|\nabla v|^2\mathrm{d}x\mathrm{d}y\Big/\int_D v^2\mathrm{d}x\mathrm{d}y=\int_D|\nabla u|^2\Big/\int_D(u^2+c_0^2)\mathrm{d}x\mathrm{d}y<\int_D|\nabla u|^2\mathrm{d}x\mathrm{d}y/\int_D|u|^2\mathrm{d}x\mathrm{d}y$，由此易见 $\lambda_2\geqslant\lambda_1$.

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